DISCONTINUOUS APPROXIMATION OF VISCOUS TWO-PHASE FLOW IN HETEROGENEOUS POROUS MEDIA

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ABSTRACT. Two phase flow in porous media is well known for its high importance in many of the industrial and engineering applications like petroleum reservoir, sedimentation process, water management in polymer electrolyte fuel cells, environmental remediation etc. By the two phase flow we mean the simultaneous flow of two fluids (for instance oil and water) and a porous medium is a substance that contains pores, or spaces between solid material through which liquid or gas can pass. This flow process is modelled by a set of partial differential equations of which the exact solution could be used to describe the flow process or to predict the behavior of the flow in advance. Unfortunately for several reasons its hard to obtain such solutions, however on the other hand the entire industry increasingly relies on the numerical simulation of the model equations. Apparently the complexities in the flow process make it hard to conduct an accurate numerical simulation of the underlying physical model. In this talk we present the Runge-Kutta Discontinuous Galerkin (RKDG) and Discontinuous Finite Volume Element (DFVE) methods which are applied to a coupled flow-transport problem describing the immiscible displacement of a viscous incompressible fluid in a non-homogeneous porous medium. The model problem consists of a nonlinear pressure-velocity equation assuming Brinkman flow, coupled to a non-linear hyperbolic equation governing the mass balance (saturation equation). The mass conservation properties inherent to finite volume-based methods motivate a DFVE scheme for the approximation of the Brinkman flow in combination with a RKDG method for the spatio-temporal discretization of the saturation equation. Also we present the stability of the scheme for the saturation equation together with a few numerical results.

Keywords:Discontinuous Galerkin method, discontinuous flux, discontinuous finite volume element method, conservation law, two-phase flow, DFLU flux.

Mathematics Subject Classifications (2010):76S05, 65M08, 65M60, 65M12.

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