FULLY DISCRETE $\mathbf{A} - \phi$ FINITE ELEMENT METHOD FOR MAXWELL'S EQUATIONS WITH NONLINEAR CONDUCTIVITY

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ABSTRACT. This talk is referred to the study of a fully discrete $\mathbf{A} - \phi$ finite element scheme to solve nonlinear Maxwell's equations based on backward Euler discretization in time and nodal finite elements in space. The nonlinearity is due to a field-dependent conductivity with the power-law form $|\mathbf{E}|^{\alpha-1}, 0 < \alpha < 1$. The system under study is hyperbolic and due to the nonlinear conductivity it lacks strong estimates of the second time derivative. We design a nonlinear time-discrete scheme for approximation in suitable function spaces. We show the well-posedness of the problem, prove convergence for our semidiscrete scheme based on boundedness of the second derivative in the dual space and derive the error estimate. The convergence of the nonlinear term is based on the Minty-Browder technique. We also discuss the error estimate for the fully discretized problem and support the theoretical result by some numerical experiments.

Keywords: nonlinear Maxwell's equations; $\mathbf{A} - \phi$ method; nodal elements; convergence; error estimates.

Mathematics Subject Classifications (2000): 65N30, 35C15, 35Q60

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