VARIATIONAL PRINCIPLE FOR THE BIDOMAIN ELECTROPHYSIOLOGY MODEL: THEORY AND NUMERICAL APPLICATIONS

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ABSTRACT. The computer simulation of the heart has recently attracted great attention from the scientific computing community, as the mathematical models governing the electrophysiology and electromechanical behavior of active tissue are amenable to numerical methods typically employed in non-linear solid mechanics. While many works now address whole heart simulations with detailed subject-specific 3D geometries using complex biophysical ionic models, only a handful of attempts have been made to study the existence and uniqueness of the numerical solutions and the convergence of numerical methods employed in cardiac simulations [1]. In this work, we build on the variational principle presented in [2] and present a saddle-point variational principle for the general time-discretized bidomain model of cardiac electrophysiology. Using results from variational analysis [3], we derive bounds on the time-step size in order for the semi-discrete problem to have a unique solution. We further show that the saddle point problem can be effectively expressed as a minimization principle, after pointwise maximization on the recovery variables. Such minimization principle is amenable to Rayleigh-Ritz finite-element analysis. The time-step bounds guarantee the strict convexity of the objective function, thus ensuring the convergence of gradient-descent methods in the solution of the resulting nonlinear system. We also show that the FitzHugh-Nagumo model conforms to the proposed variational framework proposed in this work. The numerical implications of the proposed theory are demonstrated by way of numerical examples of simple 2D cardiac wedges and 3D biventricular activation-repolarization sequences.

Keywords: Cardiac electrophysiology, bidomain model, variational principles, non-linear finite element analysis

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