STRONG-STABILITY-PRESERVING ADDITIVE LINEAR MULTISTEP METHODS

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Abstract. The concept of strong-stability-preserving (SSP) linear multistep methods is extended to semi-discretized problems for which the upwind- and downwind-biased operators have different stiffness properties. Generalized downwind methods are introduced and their monotonicity properties are investigated. Given the admissible step size that preserves monotonicity when the upwind and downwind-biased discretizations are coupled with forward Euler, we demonstrate the existence of optimal generalized downwind methods with arbitrary order of accuracy. Examples exhibit the usefulness of these methods.

We also consider additive semi-discretizations of partial differential equations that contain terms with different stiffness properties and we study the monotonicity properties of additive linear multistep methods. For a fixed number of steps and order of accuracy, we show that additive SSP methods attain the same time-step restriction as the relevant (non-additive) SSP linear multistep methods, regardless of the stiffness of the problem.

Keywords: strong stability preservation, monotonicity, linear multistep methods, additive methods, time integration

Mathematics Subject Classifications (2010): Primary, 65M20; Secondary, 65L06

References

[1] H. W. J. Lenferink. Contractivity preserving explicit linear multistep methods. *Numerische Mathematik*, 55(2):213-223, 1989.

[2] H. W. J. Lenferink. Contractivity-preserving implicit linear multistep methods. *Mathematics of Computation*, 56(193):177-199, 1991.

[3] D. I. Ketcheson. Computation of optimal monotonicity preserving general linear methods. *Mathematics of Computation*, 78(267):1497-1513, 2009.

[4] D. I. Ketcheson. Step sizes for strong stability preservation with downwind-biased operators. SIAM *Journal on Numerical Analysis*, 49(4):1649-1660, 2011.

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