

# A SCHEME WITH DISCONTINUOUS AND CONFORMING SPACES FOR THE WAVE EQUATION

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**ABSTRACT.** Using the classical work of Friedrichs, certain boundary value problems can be posed on Hilbert spaces normed with a graph norm. Functions in such spaces arising from advective problems are found to have a weak continuity property on tent-shaped domains, where inflow and outflow boundary meet. In this setting, we present a locally implicit space-time finite element method for solving advective problems on tent-shaped domains, with spaces that conform to the continuity property. The scheme uses tent pitched meshes that form an advancing space-time front. As a model example, we consider a one-dimensional wave propagation problem, the conditions needed to have a well posed formulation, and some numerical results.

**Keywords:** hyperbolic, wave equation, inflow, trace, space-time element, weak formulation, boundary operator.

**Mathematics Subject Classifications (2010):** code 35L50, 65M60

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