CONTINUOUS, MIXED AND PRIMAL HYBRID FINITE ELEMENT METHODS FOR ELLIPTIC PROBLEMS WITH hp-ADAPTIVITY: NUMERICAL COMPARATIVE STUDIES

PHILIPPE R. B. DEVLOO, CRISTIANE O. FARIA, AGNALDO M. FARIAS, SÔNIA M. GOMES, ABIMAEL F. D. LOULA, AND SANDRA M. C. MALTA

ABSTRACT. In this work, different finite element formulations for elliptic problems are implemented and compared in terms of accuracy versus number of required degrees of freedom, and CPU time. The studied formulations are: a) the H^1-conforming Galerkin method, by using finite element spaces with hierarchical polynomial bases, as described in [3]; b) the classical Mixed Formulation by using $\mathbf{H}(div)$-conforming approximation spaces, as described in [2]; c) the DG-SIP method (Symmetric Interior Penalty formulation); d) The Primal Stabilized Hybrid Method (SHDG) [4]; and e) the SHDG (Stabilized Hybrid Locally Discontinuous but Globally Continuous) formulation [1]. Excepting the DG-SIP formulation, all the other cases allow static condensation formulations, with reduced global system of equations. Two test problems are simulated. One has smooth solution in a 3-dimensional region, and is solved using uniform Cartesian meshes. The other one is 2-dimensional, and applies hp-refined meshes adapted to an internal layer with strong gradients close to it. The different formulations are compared in terms of the L^2-norm of the approximation errors in the solution and in its gradient, number of equations to be solved, with and without static condensation, and CPU times required for the simulations.

Keywords: finite elements; conforming approximations, discontinuous approximations, hybrid methods, $\mathbf{H}(div)$ spaces, hp refinement

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FEC, UNIVERSIDADE ESTADUAL DE CAMPINAS, CAMPINAS-SP, BRAZIL
E-mail address: phil@fec.unicamp.br

IME, UNIVERSIDADE ESTADUAL DO RIO DE JANEIRO, RIO DE JANEIRO-RJ, BRAZIL
E-mail address: cofaria@ime.uerj.br

IMECC, UNIVERSIDADE ESTADUAL DE CAMPINAS, CAMPINAS-RJ, BRAZIL
E-mail address: agnaldofarias.mg@gmail.com, soniag@ime.unicamp.br

CMA, LABORATÓRIO NACIONAL DE COMPUTAÇÃO CIENTÍFICA, PETRÓPOLIS-RJ, BRAZIL
E-mail address: aloc@lncc.br, smcm@lncc.br