AN ADER SCHEME WITH A NEW SOLVER FOR THE GENERALIZED RIEMANN PROBLEM AND LOCAL SPACE-TIME DG FLUX INTEGRATION

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Abstract. In high order shock-capturing methods for hyperbolic conservation laws of generalized Godunov type, the solution of the conservation law is represented at each time-step by a piecewise smooth function (say, a WENO polynomial). The resulting initial value problem with piecewise smooth but discontinuous initial data is called the *generalized Riemann problem (GRP)*. A popular strategy for constructing high order methods is then to use a *direct* solution method of the generalized Riemann problem as a building block for the flux computation (as opposed to the *evolution in the small* framework).

A highly successful variant of this approach is the ADER method with the GRP solver of Toro and Titarev [1]. While this approach achieves good results in practise, the solver can encounter difficulties when the initial data contains very large jumps, [2]. This problem does not occur in the GRP solver of LeFloch and Raviart [3], which is based on the Rankine-Hugoniot conditions. On the other hand, to satisfy the Rankine-Hugoniot conditions, for *each wave* in the solution an amount of work comparable to the Cauchy-Kovalevskaya procedure has to be carried out.

We propose a new solver for the generalized Riemann problem based on a simplified version of the LeFloch-Raviart solver. Contrary to the original approach, in our new solver no higher order derivatives of the flux or of the state inside rarefaction waves need to be computed. By using approximate Rankine-Hugoniot conditions, we can reduce the symbolic complexity of the LeFloch-Raviart solver substantially. For a broad set of test problems, the new solver gives very accurate results, even when the jump in the initial data is large.

Moreover, we extend the local space-time DG method of Dumbser, Enaux and Toro [4] for the time integration to the direct solution framework, allowing us to avoid the Cauchy-Kovalevskaya procedure for the flux computation entirely. A new *flux expansion* ansatz is shown numerically to produce very accurate results, without introducing spurious oscillations.

Keywords: ADER schemes, generalized Riemann problems

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