NUMERICAL SIMULATION OF TSUNAMIS USING ADAPTIVE MULTIRESOLUTION DISCONTINUOUS GALERKIN SCHEMES

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Abstract. We present an adaptive strategy for solving shallow water equations by high order discontinuous Galerkin methods [1, 2] with dynamic grid adaptation including a sparse representation of the bottom topography consisting of piece-wise polynomials. The scheme is used to simulate a tsunami runup onto a complex three-dimensional beach. In order to validate our method we consider a 1 : 400 scale experiment of a runup on Okushiri Island (Japan) in 1993.

The underlying idea of our adaptive strategy is to perform a multiresolution analysis using multiwavelets on a hierarchy of nested grids. This provides information on the difference between successive refinement levels that may become negligibly small in regions where the solution is locally smooth. Applying thresholding, the data is compressed thereby triggering local grid adaptation [3, 4, 6, 5, 6]. Furthermore, this information is used as an additional indicator for limiting.

A challenge in computing approximate solutions to the shallow water equations including wetting and drying is to achieve the positivity of the water height and the well-balancing of the approximate solution. The key properties of our adaptive strategy are that it is conservative and guarantees that these properties are preserved during the refinement and coarsening steps in the adaptation process [6].

Keywords: Grid adaptivity; Multiresolution analysis; Multiwavelets; Discontinuous Galerkin; Shallow water flows; Positivity-preserving; Well-balancing; Limiters; Shock detection

Mathematics Subject Classifications (2010): 65M50, 65M60, 35L65, 65T60

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