A NEW CLASS OF GALERKIN FEM WAVE PROPAGATION MODELS

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ABSTRACT. We consider the Helmholtz acoustic wave propagation model in a bounded media Ω with an inhomogeneous impedance boundary condition on $\partial\Omega$. It is well known that the standard Galerkin variational formulation of the Helmholtz partial differential equation (PDE) in $H^1(\Omega)$ is indefinite for large wavenumbers, while the Helmholtz PDE is not indefinite [3]. The lack of coercivity (indefiniteness) in the standard Galerkin problem and associated finite element method (FEM) models is one of the major difficulties for simulating wave propagation models using iterative methods.

The concept used in some literature of terming the Helmholtz equation indefinite was questioned in a recent SIAM Review article [3]. For the constant coefficient Helmholtz equation case, it was theoretically demonstrated in [3] that a non-standard Galerkin approach can produce a sign-definite variational formulation of the wave propagation model in homogeneous media.

However, the authors of the theoretical article [3] also questioned the practical use of their new sign-definite formulation even for the constant coefficient Helmholtz equation with impedance boundary condition. In particular, the key open issue mentioned in [3] is: *Whether the new formulation can alleviate some of this difficulty remains to be seen and will require a detailed, separate investigation.*

Our investigation begins with addressing this key issue of a specific parameter based formulation in [3]. Through various computer simulations, we provide a concrete answer that the sign-definite formulation analyzed in [3] does not alleviate the key difficulty of reducing the GMRES iterations of the associated FEM model if the choice of parameter that facilitates the proof of sign-definiteness of the formulation in [3] is used.

We subsequently develop a new class of Galerkin FEM wave propagation models in both homogeneous and heterogeneous media [1, 2]. In this talk we discuss our new sign-definite Galerkin FEM formulation, simulation, and analysis.

Keywords: Helmholtz equation, Galerkin variational formulation, sign-definite

Mathematics Subject Classifications (2010): 35L05, 65M60, 68U20

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