

# ON THE COUPLING OF DPG AND BEM

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ABSTRACT. In this talk we present results of our recent work [2]: We develop and analyze strategies to couple the discontinuous Petrov-Galerkin method with optimal test functions to

- (i) least-squares boundary elements and
- (ii) various variants of standard Galerkin boundary elements.

The procedure (i) is somehow a natural approach, because the DPG method can also be equivalently written as a least-squares problem. However, the implementation involves, besides the computation of discrete boundary integral operators, the evaluation of non-local norms, which is not needed for the methods (ii). The derivation of the procedures (ii) relies on either one or both equations of the Calderón system and their analysis makes use of various ideas resp. results for the coupling of FEM and BEM [1, 3, 4]. The stability of these methods hinges on an additional parameter, i.e., a scaling of the trial-to-test operator or the test functions. Nevertheless, numerical experiments indicate that the stability does not depend on this parameter.

An essential feature of the methods (i)–(ii) is that, despite the use of boundary integral equations, optimal test functions have to be computed only locally.

We apply our findings to a standard transmission problem in full space and present numerical examples to validate our theory.

**Keywords:** transmission problem, DPG method with optimal test functions, boundary elements, least-squares method, coupling, ultra-weak formulation, Calderón projector

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