FIXED POINT STRATEGIES FOR MIXED VARIATIONAL FORMULATIONS OF THE STATIONARY BOUSSINESQ PROBLEM

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ABSTRACT. In this talk we present the analysis of two new mixed variational formulations of the Boussinesq problem in steady state for describing incompressible non-isothermal fluid flows. More precisely, we introduce mixed-primal and fully-mixed approaches, both of them suitably augmented with Galerkin-type equations, and show that the resulting continuous and discrete systems can be rewritten, equivalently, as fixed-point operator equations. Then, classical arguments from linear and nonlinear functional analysis are employed to conclude that they are well-posed. The convergence of the respective Galerkin schemes is showed by employing Strang-type estimates, and with optimal order for feasible choices of subspaces. Several numerical examples support the theoretical results and illustrate the performance of our finite element methods.

Keywords: Boussinesq problem, augmented mixed-primal formulation, augmented fully-mixed formulation, fixed-point operator equation, Strang-type estimate, finite element method.

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