HDG METHODS FOR FRACTIONAL DIFFUSION

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ABSTRACT. We report on the study of the hybridizable discontinuous Galerkin method for num erically solving fractional diffusion equations of order $-\alpha$ with $-1 < \alpha < 0$ carried out in [**?**]. For exact time-marching, we derive optimal algebraic error estimates assuming that the exact solution is sufficiently regular. Thus, if for each time $t \in [0, T]$ the approximations are taken to be piecewise polynomials of degree $k \geq 0$ on the spatial domain Ω, the approximations to u in the $L_\infty\big(0, T; L_2(\Omega)\big)$-norm and to ∇u in the $L_\infty\big(0, T; \mathbf{L}_2(\Omega)\big)$-norm are proven to converge with the rate h^{k+1}, where h is the maximum diameter of the elements of the mesh. Moreover, for $k \geq 1$ and quasi-uniform meshes, we obtain a superconvergence result which allows us to compute, in an elementwise manner, a new approximation for u converging with a rate of $\sqrt{\log(Th^{-2/(\alpha+1)})}\, h^{k+2}$. These results hold uniformly in $\alpha \in (-1, 0]$ provided the exact solution is smooth. Numerical results validating the theoretical results are displayed.

Keywords: Anomalous diffusion, sub-diffusion, discontinuous Galerkin methods, hybridization, convergence analysis, superconvergence.

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