A CHARACTERISTIC-BASED CFL CONDITION FOR THE DISCONTINUOUS GALERKIN METHOD ON TRIANGULAR MESHES

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ABSTRACT. When the discontinuous Galerkin (DG) method is applied to hyperbolic problems in two dimensions on triangular meshes and paired with an explicit time integration scheme, an exact CFL (Courant-Friedrichs-Lewy) condition is not known. The stability condition which is most usually implemented involves scaling the time step by the smallest radius of the inscribed circle in every cell. However, this is known to not provide a tight bound on the largest possible time step in some cases. The main difficulty in finding a suitable CFL number is its dependence on the orientation of a triangle with respect to the direction of the flow. Usually, the smallest CFL over all directions is taken, which is clearly not optimal. By using an approach introduced in [2], we find a natural scaling of the spectrum of the DG spatial operator by a parameter h_j, which can be seen to be the width of the cell Ω_j along the characteristic direction of flow. We show that this parameter h_j incorporates 95% of the variation of the spectrum with respect to the orientation of Ω_j to flow direction. We use this parameter to propose a new CFL condition. For a classical paring of the DG method with a degree p basis and an explicit Runge-Kutta scheme of order $p+1$, we show that a CFL number equal to $1/(2p+1)(1+(2/(2p+2))^2)$ is within 5% accurate for $p \leq 10$.

We show through several numerical examples that we are able to select larger stable time steps than usually obtained. The gain is the largest for anisotropic meshes aligned with the flow direction.

Keywords: Discontinuous Galerkin Method, stability, CFL condition

Mathematics Subject Classifications (2010): 65M60

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