A FULLY-MIXED FORMULATION FOR THE NAVIER–STOKES/DARCY COUPLED PROBLEM WITH NONLINEAR VISCOSITY

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Abstract. We propose and analyse an augmented mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by a class of nonlinear Navier–Stokes and the linear Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers–Joseph–Saffman law. We apply dual-mixed formulations in both domains, and the nonlinearity involved in the Navier–Stokes region is handled by setting the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as the associated Lagrange multipliers. Furthermore, since the convective term in the fluid forces the velocity to live in a smaller space than usual, we augment the variational formulation with suitable Galerkin type terms. The resulting augmented scheme is then written equivalently as a fixed point equation, so that the well-known Schauder and Banach theorems, combined with classical results on bijective monotone operators, are applied to prove the unique solvability of the continuous and discrete systems. Finally, several numerical results illustrating the good performance of the augmented mixed finite element method and confirming the theoretical rates of convergence are reported.

Keywords: Navier–Stokes problem, Darcy problem, stress-velocity formulation, fixed point theory, mixed finite element methods, a priori error analysis.

Mathematics Subject Classifications (2010): 65N30, 65N12, 80A20, 76R05, 76D07

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