A COMPACT-STENCIL SCHEME OF ORDER $\frac{3}{2}$ ON POLYHEDRAL MESHES FOR ADVECTION-REACTION EQUATIONS

P. CANTIN, E. BURMAN, AND A. ERN

Abstract. Motivated by the recent CDO (compatible discrete operator) framework, see [1, 2], our goal is to develop and analyze numerical schemes that support general meshes with polyhedral cells and nonmatching interfaces. Recently, a CDO scheme of order $\frac{1}{2}$ with degrees of freedom attached to polyhedral mesh vertices has been proposed for the scalar advection-reaction and advection-diffusion problems on general meshes, see [3]. In the present work, we focus on the advection-reaction equation, and we devise and analyze a new scheme of order $\frac{3}{2}$. The key idea is to consider degrees of freedom attached to mesh vertices and to cell centers (as for VAG schemes [4]). Then, taking inspiration from the recent analyze of finite element methods stabilized by local continuous interior penalty [5], the present scheme is stabilized by penalizing the gradient jump on some internal faces inside each mesh cell. Well-posedness of the scheme follows from an inf-sup stability condition, and error estimates of order $\frac{3}{2}$ are inferred for smooth solutions by bounding the consistency error. Numerical results are presented on three-dimensional polyhedral meshes [6], and the benefit (or not) of static condensation to eliminate degrees of freedom attached to cells is discussed.

Keywords: Polyhedral meshes, advection-reaction, stabilization, static condensation.

Mathematics Subject Classifications (2010): 65N15, 65N30, 65Zxx

References

[1] J. Bonelle, and A. Ern. Analysis of Compatible Discrete Operator Schemes for Elliptic Problems on Polyhedral Meshes. *ESAIM : Mathematical Modelling and Numerical Analysis*, 48(2):553-581, 2014.

[2] J. Bonelle, and A. Ern. Analysis of Compatible Discrete Operator Schemes for Stokes Problems on Polyhedral Meshes. *IMA Journal of Numerical Analysis*, doi:10.1093/imanum/dru051, 2014.

[3] P. Cantin, and A. Ern. Vertex-based Compatible Discrete Operator Schemes on Polyhedral Meshes for Advection-Diffusion Equations. *Hal-01141290*, 2015.

[4] R. Eymard, C. Guichard, and R. Herbin. Small-stencil 3D schemes for diffusive flows in porous media. *ESAIM : Mathematical Modelling and Numerical Analysis*, 46(2):265-290, 2011.

[5] E. Burman, and F. Schieweck. Local CIP stabilization for composite finite elements. *In progress*, 2015.

[6] R. Eymard, G. Henry, R. Herbin, F. Hubert, R. Klofkorn, and G. Manzini. 3D Benchmark on Discretization Schemes for Anisotropic Diffusion Problems on General Grids. *Proceedings of Finite Volumes for Complex Applications VI*, 895-930, 2011.

Université Paris-Est, CERMICS (ENPC), 77455 Marne la Vallée Cedex 2, France
E-mail address: pircantin@gmail.com

Dept. of Math., University College London, Gower Street, London, United Kingdom.
E-mail address: e.burman@ucl.ac.uk

Université Paris-Est, CERMICS (ENPC), 77455 Marne la Vallée Cedex 2, France
E-mail address: ern@cermics.enpc.fr