A COMPACT-STENCIL SCHEME OF ORDER $\frac{3}{2}$ ON POLYHEDRAL MESHES FOR ADVECTION-REACTION EQUATIONS

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Abstract. Motivated by the recent CDO (compatible discrete operator) framework, see [1, 2], our goal is to develop and analyze numerical schemes that support general meshes with polyhedral cells and nonmatching interfaces. Recently, a CDO scheme of order $\frac{1}{2}$ with degrees of freedom attached to polyhedral mesh vertices has been proposed for the scalar advection-reaction and advection-diffusion problems on general meshes, see [3]. In the present work, we focus on the advection-reaction equation, and we devise and analyze a new scheme of order $\frac{3}{2}$. The key idea is to consider degrees of freedom attached to mesh vertices and to cell centers (as for VAG schemes [4]). Then, taking inspiration from the recent analyze of finite element methods stabilized by local continuous interior penalty [5], the present scheme is stabilized by penalizing the gradient jump on some internal faces inside each mesh cell. Well-posedness of the scheme follows from an inf-sup stability condition, and error estimates of order $\frac{3}{2}$ are inferred for smooth solutions by bounding the consistency error. Numerical results are presented on three-dimensional polyhedral meshes [6], and the benefit (or not) of static condensation to eliminate degrees of freedom attached to cells is discussed.

Keywords: Polyhedral meshes, advection-reaction, stabilization, static condensation.

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References


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