AN ADAPTIVE CHOICE OF PRIMAL CONSTRAINTS FOR BDDC DOMAIN DECOMPOSITION ALGORITHMS

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Abstract. An adaptive choice based on parallel sums for the primal space of BDDC [1] deluxe methods [2] is analyzed. The primal constraints of a BDDC algorithm provide the global, coarse part of such a preconditioner and is of crucial importance for obtaining rapid convergence of these preconditioned conjugate gradient methods for the case of many subdomains.

For problems in three dimensions, there is a need to develop algorithms and results for equivalence classes with three or more elements, e.g., subdomain edges. For this purpose, parallel sums for general equivalence classes are considered. The use of parallel sums for equivalence classes with two elements (subdomain faces) has proven very successful; see [3].

An upper bound of the square of the norm of a jump operator P_D acting on the elements in a product space related to the subdomains is derived; it has been known that such a bound provides an estimate of the condition number of the BDDC algorithm; see [4]. This bound is given in terms of parallel sums of single Schur complements and sums of other Schur complements. Hence, generalized eigenvalue problems with parallel sums related to the faces and edges of the subdomains are formulated. A few eigenvectors associated with the smallest eigenvalues are selected and they generate a primal constraint. These generalized eigenvalue problems are defined in terms of the relevant Schur complements and Schur complements of these Schur complements associated with a minimal energy extension, e.g., from a subdomain edge of a three-dimensional finite element problem.

Numerical results for elliptic problems verify the performance of the algorithm, using a series of experiments with regular subdomains as well as subdomains generated by a METIS mesh partitioner. There is also fast convergence for problems with a quite irregular coefficient inside the subdomains.

Keywords: domain decomposition, BDDC deluxe preconditioners, adaptive primal constraints, elliptic problems.

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