A MIXED VIRTUAL ELEMENT METHOD FOR QUASI-NEWTONIAN STOKES FLOW

ERNESTO CÁCERES, GABRIEL N. GATICA, AND FILÁNDER A. SEQUEIRA

Abstract. In this work, we introduce and analyze a virtual element method (VEM) for a mixed variational formulation of a class of nonlinear Stokes models arising in quasi-Newtonian fluids. The pseudostress, the pressure and the velocity are the original unknowns, whereas the velocity gradient is introduced as an auxiliary variable, and the pressure is computed via a postprocessing formula. An augmented formulation showing a single saddle point structure is then obtained and its corresponding well-posedness is stablished by using known results from nonlinear functional analysis. Following the basic principles and ideas from previous results on a mixed-VEM for the Stokes problem, we define the virtual finite element subspaces to be employed, introduce the associated interpolation operators, and provide the respective approximation properties. On the other hand, we propose a new local projector onto a suitable space of polynomials, which takes into account the main features of the continuous solution and allows the explicit integration of the terms involving the deviatoric tensors. The uniform boundedness of the resulting family of local projectors is established and its approximation properties are also derived. In addition, we show that the corresponding nonlinear discrete scheme is also uniquely soluble, and derive the associated a priori error estimates for the virtual solution as well as for the fully computable projection of it. Furthermore, we introduce a second element-by-element postprocessing formula for the pseudostress, which yields an optimally convergent approximation of such unknown with respect to the broken $\mathbb{H}(\mathbf{div})$-norm. Finally, several numerical results illustrating the good performance of the method and confirming the theoretical rates of convergence are presented.

Keywords: Stokes equations, virtual element method, a priori error analysis

Mathematics Subject Classifications (2010): 65N30, 65N12, 65N15, 76D07

References

[1] L. Beirão da Veiga, F. Brezzi, A. Cangiani, L.D Marini, G. Manzini, A. Russo. Basic principles of virtual elements methods. Math. Models Methods Appl. Sci. 23 (2013), no. 1, 199–214.

[2] F. Brezzi, R.S. Falk, L.D. Marini. Basic principles of mixed virtual element methods. ESAIM Math. Model. Numer. Anal. 48 (2014), no. 4, 1227–1240.

[3] E. Cáceres, G.N. Gatica. A mixed virtual element method for the pseudostress-velocity formulation of the Stokes problem. Preprint available at ftp://ftp.ci2ma.udec.cl/pub/ci2ma/pre-publicaciones/2015/pp15-05.pdf

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile
E-mail address: ecaceresv@udec.cl

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile
E-mail address: ggatica@ci2ma.udec.cl

CI^{2}MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile
E-mail address: fsequeira@ci2ma.udec.cl