FINITE DIFFERENCE WENO SCHEMES FOR MULTI-DIMENSIONAL MULTIPHASE FLOWS IN POROUS MEDIA

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Abstract. Mathematical models for multiphase flow processes in porous media under vertical equilibrium have been used since the end of XIX century in many different physical situations as water filtration [5], enhanced oil recovery [1], etc.

It is the primary purpose of this contribution to introduce the use of finite difference WENO schemes to multiphase flow problems in porous medium for multi-dimensional problems, following the work developed by Donat et al. in [2, 3, 4], where a progressive evolution towards the application of WENO schemes to one-dimensional multiphase porous media flow processes is performed.

In the one-dimensional setting the expression of the gradient of the pressure appearing in the constitutive mathematical model equations can be explicitly derived from the equations (see [3]), but that is not possible in a multi-dimensional frame, which encourages us to develop some new methodology to apply finite difference conservative WENO schemes to a multi-dimensional version of this problem. The key idea behind the proposed numerical technique is to define a compatible discretization for the fluxes of the convective term in order to maintain their divergence-free character not only in the continuous setting but also in the discrete setting, assuring the conservation of the sum of the saturations through time evolution.

Keywords: Finite difference WENO schemes, Porous media, Multiphase flow, Vertical equilibrium

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