A RANDOM SAMPLING APPROACH FOR A FAMILY OF TEMPLE-CLASS SYSTEMS OF CONSERVATION LAWS

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Abstract. Several applications, including the Aw-Rascle-Zhang traffic model [1, 6] and a model of sedimentation of small particles in a viscous fluid [2], give rise to nonlinear 2×2 systems of conservation laws that are governed by a single scalar system velocity, which is associated to a scalar flux function. Such systems are of the Temple class since rarefaction wave curves and Hugoniot curves coincide. Moreover, one characteristic field is genuinely nonlinear (with the exception of a manifold of inflection points of the scalar flux function) and the other is linearly degenerate. For systems of this family, there are two well-known problems. The vacuum state, which may form naturally even from positive initial data, gives rise to potential problems of non-uniqueness and instability. This is resolved by the introduction of two alternative solution concepts of the Riemann problem. The other problem are spurious oscillations produced by Godunov's method near contact discontinuities. This behaviour actually arises with many standard conservative schemes since the numerical solution invariably leaves the invariant region of the exact solution. It is demonstrated that a strategy consisting of alternating between averaging (Av) and remap steps similar to the approach by Chalons and Goatin [3] generates numerical solutions that satisfy an invariant region property (in contrast to, for instance, Godunov's method). For the case that the remap step is done by random sampling (RS), techniques due to J. Glimm [4] and R.J. LeVeque and B. Temple [5] are combined to prove that the resulting statistically conservative Av-RS scheme converges to a weak solution. Numerical examples illustrate the performance of the Av-RS scheme, and its superiority over Godunov's method in terms of accuracy and resolution.

Keywords: Averaging-remap schemes, anti-diffusive, statistically conservative, convergence, Aw-Rascle-Zhang traffic model, sedimentation.

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