NUMERICAL SOLUTION OF EULER EQUATIONS WITH SOURCE TERMS

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Abstract. Numerical solution of nonlinear hyperbolic conservation laws by using finite volume discretizations and approximate Riemann solvers is now a well established subject (see, for instance, [4]). However, if sources are present, the problem is more involved and it is now a very active research topic (see, for instance, [3], [2] and the references therein). In the present work, a finite volume scheme for the numerical solution of a mathematical model for non-isothermal non-adiabatic compressible flow of mixture of perfect gases in a pipeline is introduced. Unlike standard Euler equations, the model takes into account wall friction, variable height along the pipeline, and heat transfer between the pipe and its environment. Since all these terms are sources, in order to get a well-balanced scheme they have been discretized by using some upwinding similar to the one in the flux term, following the methodology introduced in [1]. The performance of the overall method has been shown for some numerical tests. The final goal, not included in this presentation, is to consider a network with several connected pipelines, compressors, valves, etc., as those employed for natural gas transport.

Keywords: Gas flow in pipelines, finite volume method, upwind schemes, nonlinear hyperbolic systems with sources, variable topography

Mathematics Subject Classifications (2010): 35L60, 65M08, 76N15.

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