# STABLE PERFECTLY MATCHED LAYERS FOR COLD PLASMAS IN STRONG MAGNETIC FIELDS

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ABSTRACT. We consider the problem of wave propagation in cold magnetized plasmas in unbounded domains. We concentrate on a simplified cold plasma model, which in the frequency domain reads

$$\operatorname{curl}\operatorname{curl}\mathbf{E} - \frac{\omega^2}{c^2}\epsilon(\omega)\mathbf{E} = 0,\ \epsilon(\omega) = \operatorname{diag}\left(1,\ 1,\ 1-\frac{\omega_p^2}{\omega^2}\right),\ \omega_p > 0. \tag{1}$$

To bound the computational domain, we suggest using the perfectly matched layer (PML) technique. It is well-known [1, 2] that the Bérenger's PMLs can exhibit instabilities in anisotropic and/or dispersive media, due to the presence of backward propagating modes [2]. This work deals with the construction of stable PMLs for the model (1).

First we consider a simplified 2D case ($x = \text{const}$, TE mode). The Bérenger's PML in the direction $z$ is stable. In the direction $y$ all the waves propagate backwards for $\omega < \omega_p$ and forward otherwise. Therefore, in this case we suggest to use an improved PML proposed in [1], which is particularly well-suited for problems that have for a fixed frequency only forward or only backward propagating modes.

In 3D, the situation is different. While in the direction $z$ the Bérenger's PML is still stable, and in the directions $x, y$, for $\omega > \omega_p$ there are only forward propagating modes, for $\omega < \omega_p$ there exist simultaneously forward and backward propagating waves. Therefore, the method of [1] is no longer directly applicable. To cope with this difficulty, we suggest to make use of a structure of the dispersion relation in three dimensions. More precisely, it can be represented as a product of two dispersion relations. The first one, $F_\omega(\omega, \mathbf{k})$, is the dispersion relation for the isotropic non-dispersive Maxwell equations. The second one, $F_{2D}(\omega, \mathbf{k})$, resembles the dispersion relation for the two-dimensional TM system (1). Therefore, it is possible to perform the splitting of (1) into two systems, one with the dispersion relation $F_\omega(\omega, \mathbf{k})$, and thus having only forward propagating modes, and another one with the dispersion relation $F_{2D}(\omega, \mathbf{k})$, and thus behaving like the two-dimensional case of (1). This allows to use the Bérenger's PML for the first system, and the 2D-cold-plasma-like PML for the second one. The stability of new PMLs is demonstrated with the help of theoretical and numerical arguments.

**Keywords**: perfectly matched layers, cold plasma, Maxwell equations

**Mathematics Subject Classifications (2010)**: 65M12, 65Z05

## REFERENCES

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