TIME SPLITTING WITH FINITE ELEMENT METHOD FOR SOLVING A NONLINEAR SCHRÖDINGER EQUATION

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Abstract. Ultracold gases condensation is modelled by a many body Hamiltonian. Using The Mean Field Theory, all particles move with a single wave function ψ preserving the number of particles

$$\int_{\mathcal{R}^3} |\psi_H(x,t)|^2 dx = 1.$$

The evolution for ultracold gases is given by the nonlinear Schrödinger equation (NLSE)

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\Delta\psi(x,t) + V(x)\psi(x,t) + g_B\psi(x,t)^3,$$

where $g_B = \frac{4\pi\hbar^2}{m}a_s$ is the attractive ($g_B < 0$) and repulsive ($g_B > 0$) parameter of the different atom species.

For solving NLSE, different numerical techniques have been used. In [2], the authors compare different time and space approaches: time splitting pseudospectral, Crank-Nicolson and Finite Differences, Relaxed Finite Differences, Semi-implicit Finite Differences, and Time Splitting Finite Differences. Also, they summarize what methods preserve mass and energy, two very important properties for the physical interest in these methods. In Table 2 [2], we observe that 1D NLSE is solved for these five approaches. All space finite difference methods at fixed time $t = 5$ have same errors for different space steps except for TSSP, which is very precise. On the other hand, in Table 3 for time steps, errors are similar. Thus, Time Splitiing is better for controlling error, but energy is not conserved. On the other hand, Finite Differences are less precise but energy preserving.

We will study different approaches for preserving energy in Time Splitting methods [1] combined with the good properties of the adaptive Finite Element Methods to get a fast convergent method preserving mass and controling the energy losses. We compare this method with high order time splitting methods given in [3] and TSFD for 1D and 2D problems studying the error behavior and finding new and fast strategies.

Keywords: nonlinear Schrödinger equation, time splitting, finite element methods, mass conservation, energy conservation

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