A MIXED-PRIMAL FINITE ELEMENT APPROXIMATION OF A SEDIMENTATION-CONSOLIDATION SYSTEM

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Abstract. This work is devoted to the mathematical and numerical analysis of a strongly coupled flow and transport system typically encountered in continuum-based models of sedimentation-consolidation processes. The model focuses on the steady-state regime of a solid-liquid suspension immersed in a viscous fluid within a permeable medium, and the governing equations consist in the Brinkman problem with variable viscosity, written in terms of Cauchy pseudo-stresses and bulk velocity of the mixture; coupled with a nonlinear advection – nonlinear diffusion equation describing the transport of the solids volume fraction. The variational formulation is based on an augmented mixed approach for the Brinkman problem and the usual primal weak form for the transport equation. Solvability of the coupled formulation is established by combining fixed point arguments, certain regularity assumptions, and some classical results concerning variational problems and Sobolev spaces. In turn, the resulting augmented mixed-primal Galerkin scheme employs Raviart-Thomas approximations of order k for the stress and piecewise continuous polynomials of order $k+1$ for velocity and volume fraction, and its solvability is deduced by applying a fixed-point strategy as well. Then, suitable Strang-type inequalities are utilized to rigorously derive optimal error estimates in the natural norms. Next, a few numerical tests illustrate the accuracy of the augmented mixed-primal finite element method, and the properties of the model. In addition, we derive two efficient and reliable residual-based *a posteriori* error estimators for our augmented mixed–primal finite element scheme.

Keywords: Brinkman equations, nonlinear transport problem, augmented mixed–primal formulation, fixed point theory, sedimentation-consolidation process, finite element methods, a priori error analysis, a posteriori error analysis.

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