ERROR ANALYSIS FOR POD APPROXIMATIONS IN A DYNAMIC PROGRAMMING FRAMEWORK

A. ALLA, M. FALCONE, AND STEFAN VOLKWEIN

Abstract. For nonlinear optimal control problems, feedback laws can be computed via the value function characterized as the unique viscosity solution to the corresponding Hamilton-Jacobi-Bellman (HJB) equation which stems from the dynamic programming approach. However, the bottleneck is mainly due to the *course of dimensionality* and HJB equations are only solvable in a relatively small dimension. Therefore, a reduced-order model is derived for the dynamical system and for this purpose we use the method of proper orthogonal decomposition (POD). We will apply these techniques to solve the infinite horizon optimal control problem for a nonlinear high-dimensional dynamical system. The resulting errors in the HJB equations are estimated via an *a-priori* error analysis, which suggests a new sampling strategy for the POD method. Numerical experiments will illustrate the proposed method.

Keywords: optimal control, dynamic programming, POD approximation, error estimates.

Mathematics Subject Classifications (2010): 49L20, 49M30, 93C20

References

[1] A. Alla, and M. Falcone. An adaptive POD approximation method for the control of advection-diffusion equations, in K. Kunisch, K. Bredies, C. Clason, G. von Winckel, (eds) *Control and Optimization with PDE Constraints,* International Series of Numerical Mathematics, **164**, Birkhäuser, Basel, 2013, 1-17.

[2] A. Alla and M. Falcone. A Time-Adaptive POD Method for Optimal Control Problems, in the Proceedings of the 1st IFAC Workshop on Control of Systems Modeled by Partial Differential Equations, **1**, 2013, 245-250.

[3] A. Alla, M. Falcone, D. Kalise An efficient policy iteration algorithm for dynamic programming equations, SIAM J. Sci. Comput., **37**(1), 2015, 181-200.

[4] A. Alla, M. Falcone and S. Volkwein. Error analysis for POD approximations of infinite horizon problems via the Dynamic Programming approach, preprint, 2015, `arXiv:1511.00265`

[5] K. Kunisch, S. Volkwein and L. Xie. HJB-POD Based Feedback Design for the Optimal Control of Evolution Problems, SIAM J. on Applied Dynamical Systems, **4**, 2004, 701-722.

Universität Hamburg
E-mail address: `alessandro.alla@uni-hamburg.de`

SAPIENZA, università di Roma
E-mail address: `falcone@mat.uniroma1.it`

University of Kostanz
E-mail address: `Stefan.Volkwein@uni-konstanz.de`