

# NONLOCAL DIFFUSIONS ON FRACTALS. QUALITATIVE PROPERTIES AND NUMERICAL APPROXIMATIONS

MARCELO ACTIS, MARILINA CARENA, AND PEDRO MORIN

ABSTRACT. We propose a numerical method to approximate the solution of a nonlocal diffusion problem on a general setting of metric measure spaces. These spaces include, but are not limited to, fractals, manifolds and Euclidean domains. We obtain error estimates in  $L^\infty(L^p)$  for  $p = 1, \infty$  under the sole assumption of the initial datum being in  $L^p$ . An improved bound for the error in  $L^\infty(L^1)$  is obtained when the initial datum is in  $L^2$ . We also derive some qualitative properties of the solutions like stability, comparison principles and study the asymptotic behavior as  $t \rightarrow \infty$ . We finally present two examples on fractals: the Sierpinski gasket and the Sierpinski carpet, which illustrate on the effect of nonlocal diffusion for different initial data.

**Keywords:** nonlocal diffusions, discretizations, space of homogeneous type, fractals

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INSTITUTO DE MATEMÁTICA APLICADA DEL LITORAL (CONICET-UNL) AND, FACULTAD DE INGENIERÍA QUÍMICA, UNIVERSIDAD NACIONAL DEL LITORAL, SANTA FE, ARGENTINA

*E-mail address:* [mactis@santafe-conicet.gov.ar](mailto:mactis@santafe-conicet.gov.ar)

INSTITUTO DE MATEMÁTICA APLICADA DEL LITORAL (CONICET-UNL) AND, FACULTAD DE HUMANIDADES Y CIENCIAS, UNIVERSIDAD NACIONAL DEL LITORAL, SANTA FE, ARGENTINA

*E-mail address:* [mcarena@santafe-conicet.gov.ar](mailto:mcarena@santafe-conicet.gov.ar)

INSTITUTO DE MATEMÁTICA APLICADA DEL LITORAL (CONICET-UNL) AND, FACULTAD DE INGENIERÍA QUÍMICA, UNIVERSIDAD NACIONAL DEL LITORAL, SANTA FE, ARGENTINA

*E-mail address:* [pmorin@santafe-conicet.gov.ar](mailto:pmorin@santafe-conicet.gov.ar)