OPERATOR PRECONDITIONING FOR THE HYPERSINGULAR OPERATOR OVER 3D SCREENS.

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Abstract. The discretization of first-kind boundary integral operators (BIO) by low-order Galerkin BEM leads to ill-conditioned linear systems on fine meshes. Consequently, iterative solvers, which become mandatory when using compression techniques to reduce the computational cost of the discrete BIOs, become slow.

Preconditioning is essential to overcome this difficulty. However, the standard "Calderón preconditioning" technique breaks down when dealing with screens. In this case, the double layer operator and its adjoint vanish. Moreover, the associated weakly singular and hypersingular operators no longer map fractional Sobolev spaces in a dual fashion.

In this presentation we propose a new Calderón preconditioner for the Hypersingular operator on 3D screens arising from the Laplacian. For its construction, we use operator preconditioning [2] and the bilinear form induced by its inverse BIO over the disk, which can be obtained from Fabrikant [1]. The novel BIO is a (modified) weakly singular operator that incorporates the distance to the boundary $\partial\Gamma$ of the screen Γ, in an analogous way to the function $M(x, y)$ in the 2D case [4, 3].

Our numerical results show the optimality of our preconditioner when applied to different screens. Furthermore, this preconditioning technique can be used on non-uniform meshes. This property poses a great advantage, as solutions of boundary integral equations on screens feature a square-root type singularity at $\partial\Gamma$, which can be resolved by refining towards the boundary.

Keywords: open surface problems, boundary integral equations, Laplace equation, operator (Calderón) preconditioning, screen problems

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