EXACTLY COMPUTABLE LAGRANGE–GALERKIN SCHEMES FOR FLOW PROBLEMS

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Abstract. We present exactly computable Lagrange–Galerkin schemes for convection-diffusion problems and Navier–Stokes problems which we recently developed [1, 2].

The Lagrange–Galerkin method, which is also called characteristics finite element method or Galerkin-characteristics method, is a powerful numerical method for flow problems, having such advantages that it is robust for convection-dominated problems and that the resultant matrix to be solved is symmetric. For the recent developments, see [3] for example.

In the Lagrange–Galerkin method we have to deal with the integration of composite functions, which is difficult to get the exact value. In real computations, numerical quadrature is usually applied to the integration to obtain approximate values, that is, the scheme is not computable exactly. It is known that the error caused from the approximation may destroy the stability result that is proved under the exact integration [1, 2, 4, 5, 6]. Here we introduce a locally linearized velocity [1, 6] and the backward Euler method in solving ordinary differential equations in the position of fluid particle. Then, the scheme becomes computable exactly, and the theoretical stability and convergence results are assured.

We show error estimates of the schemes. We also present some numerical results, which reflect these estimates and also show robust stability for high Péclet numbers or high Reynolds numbers.

Keywords: Lagrange–Galerkin scheme, finite element method, convection-diffusion problems, Navier–Stokes problems, exact integration.

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