THE ADER APPROACH FOR CONSTRUCTING NON-LINEAR SCHEMES OF ARBITRARY ACCURACY FOR SOLVING EVOLUTIONARY PDES

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Abstract. ADER is a fully-discrete approach for constructing one-step, non-linear numerical schemes of arbitrarily high order of accuracy, in both space and time, to solve evolutionary partial differential equations. The approach in the finite volume framework was first put forward in [9], where schemes for linear equations in 1D, 2D and 3D on Cartesian meshes were formulated and tested. Implementations of up to 10th order of accuracy in space and time were reported. For an introduction to ADER schemes see chapters 19 and 20 of [8]. The extension to non-linear systems relies on the solution of the generalised Riemann problem for non-linear systems and was first reported in [10] and [6]. Multidimensional extensions to non-linear systems are reported in [7]. ADER was later extended in the framework of discontinuous Galerkin finite element methods [4]. In [3] a unified finite volume/DG framework for the ADER approach is proposed. The ADER methodology has undergone numerous extensions and applications, eg [2, 1, 5, 11, 12].

Here I shall present a succinct review of the ADER approach and illustrate the performance of the schemes through some applications.

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