# A POSTERIORI ERROR ANALYSIS OF LINEAR PARABOLIC INTERFACE PROBLEMS: A RECONSTRUCTION APPROACH

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Abstract. We discuss *a posteriori* error analysis of finite element method for linear parabolic interface problems in a two-dimensional convex polygonal domain in $\mathbb{R}^2$. Interface problems arise in various applications ranging from material science to fluid dynamics when two distinct materials or fluids with different conductivities or densities or diffusion coefficients are involved. A residual-based *a posteriori* estimates for both the spatially discrete and the fully discrete approximations are considered and analyzed. The space discretization uses finite element spaces that are allowed to change in time whereas, the time discretization is based on the backward Euler approximation. The residual-based *a posteriori* error analysis relies on the approximation properties of the Clément-type interpolation operator of Scott and Zhang [3]. Due to the discontinuity of the coefficient along the interface, the solution has a lower regularity in the entire domain [1] and hence, the existing approximation results do not apply directly for interface problems. The main ingredients used in deriving *a posteriori* estimates are new Clément type interpolation estimates and an appropriate elliptic reconstruction operator introduced by Makridakis and Nochetto in [2]. We use only an energy argument to establish *a posteriori* error estimates with optimal order convergence in the $L^2(H^1)$-norm and almost optimal order in the $L^\infty(L^2)$-norm. The interfaces are assumed to be of arbitrary shape but are smooth for our purpose. Numerical results are presented to validate the asymptotic behaviour of the derived estimators.

**Keywords**: *A posteriori* error estimates, Clément-type interpolation estimates, elliptic reconstruction, parabolic interface problems.

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