DISSIPATION IN ADAPTIVE WAVELET DISCRETIZATIONS

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Abstract. Adaptive wavelet schemes for solving partial differential equations offer an attractive possibility to introduce locally refined grids, which dynamically track the evolution of the solution in scale and space. Automatic error control of the adaptive discretization, with respect to a uniform grid solution, is an advantageous feature. For a review in the context of computational fluid dynamics we refer to [4].

In this work we focus on dynamical Galerkin schemes, where the projection operator changes over time. When selecting a subset of basis functions, the projection operator is non-differentiable and an integral formulation has to be used. We will analyze the projected equations with respect to existence and uniqueness of the solution and prove that non-smooth projection operators introduce dissipation, a result which is crucial for adaptive discretizations of PDEs.

For the Burgers equation we will illustrate numerically that thresholding the wavelet coefficients, and thus changing the projection space, will indeed introduce dissipation of energy [3]. We discuss consequences for the so-called "pseudo-adaptive" simulations, where time evolution and dealising are done in Fourier space whilst thresholding is carried out in wavelet space. Applications will deal with the inviscid Burgers equation using a simple small-scale periodic filtering scheme as well as CVS filtering [2], the latter has been applied for three-dimensional turbulence in [1].

Keywords: wavelets, adaptivity, Galerkin method, dissipation

Mathematics Subject Classifications (2010): 65N30, 65N50.

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