BEM/FEM COUPLING FOR TRANSIENT ACOUSTIC SCATTERING BY PIEZOELECTRIC OBSTACLES

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ABSTRACT. We consider the interaction of an incident acoustic wave v^{inc} propagating in the unbounded region $\Omega_+ \subset \mathbb{R}^d$ and a piezoelectric solid contained in the bounded region $\Omega_- := \overline{\Omega}_+^c$. The scattered component of the acoustic wave v, the elastic displacement vector $\mathbf{u}$, and the electric potential ψ satisfy the system of PDE's

$$\Delta v = c^{-2} v_{tt} \qquad \text{in } \Omega_+ \times [0,\infty), \tag{1a}$$

$$\nabla \cdot \boldsymbol{\sigma} = \rho_\Sigma \mathbf{u}_{tt} \qquad \text{in } \Omega_- \times [0,\infty), \tag{1b}$$

$$\nabla \cdot \mathbf{D} = 0 \qquad \text{in } \Omega_- \times [0,\infty), \tag{1c}$$

$$(\mathbf{u}\cdot\boldsymbol{\nu})_t + \partial_\nu v = -\,\partial_\nu v^{inc} \qquad \text{on } \Gamma \times [0,\infty), \tag{1d}$$

$$\boldsymbol{\sigma}\,\boldsymbol{\nu} + \rho_f v_t \boldsymbol{\nu} = -\,\rho_f (v^{inc})_t \boldsymbol{\nu} \qquad \text{on } \Gamma \times [0,\infty), \tag{1e}$$

$$\mathbf{D}\cdot\boldsymbol{\nu} = \eta_d \qquad \text{on } \Gamma_N \times [0,\infty), \tag{1f}$$

$$\psi = \mu_d \qquad \text{on } \Gamma_D \times [0,\infty), \tag{1g}$$

where $\Gamma := \partial\Omega_-$ and v, v_t, $\mathbf{u}$, and $\mathbf{u}_t$ satisfy homogeneous initial conditions. The stress tensor $\boldsymbol{\sigma}$ and the electric displacement vector $\mathbf{D}$ satisfy the constitutive relations

$$\boldsymbol{\sigma} := \mathbf{C}\boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{e}\nabla\psi\,, \qquad \mathbf{D} := \mathbf{e}^\top \boldsymbol{\varepsilon}(\mathbf{u}) - \boldsymbol{\epsilon}\nabla\psi,$$

where $\mathbf{C}$ is the elastic compliance tensor, $\boldsymbol{\varepsilon}$ is the elastic strain tensor, $\mathbf{e}$ is the piezoelectric tensor, and $\boldsymbol{\epsilon}$ is the dielectric tensor. The problem is transferred into the Laplace domain and transformed into an integro-differential system. For this system, a Galerkin space discretization suitable for BEM/FEM treatment is proposed. Well posedness for the continuous and semi-discrete systems is shown simultaneously and stability bounds in terms of the Laplace parameter S are found. Full discretization is achieved using BDF2-based Convolution Quadrature for the BIE's and BDF2 time-stepping for the elastic and electric unknowns. Using the Laplace-domain stability estimates, we provide time-domian bounds with explicit time dependence and show that, for smooth enough problem data, optimal second-order convergence is achieved. Numerical experiments are given for BDF2 and Trapezoidal Rule based CQ.

Keywords: Time-Domain Boundary Integral Equations, Convolution Quadrature, Scattering, Coupling BEM/FEM, Piezoelectricity.

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