A FIRST ORDER SYSTEM LEAST SQUARES METHOD FOR THE HELMHOLTZ EQUATION

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Abstract. We present a first order system least squares (FOSLS) method for the Helmholtz equation at high wave number k, which always leads to a Hermitian positive definite algebraic system. By utilizing a non-trivial solution decomposition to the dual FOSLS problem which is quite different from that of the standard finite element methods, we give an error analysis to the hp-version of the FOSLS method where the dependence on the mesh size h, the approximation order p, and the wave number k is given explicitly. In particular, under some assumption of the boundary of the domain, the L^2 norm error estimate of the scalar solution from the FOSLS method is shown to be quasi optimal under the condition that kh/p is sufficiently small and the polynomial degree p is at least $O(\log k)$. Numerical experiments are given to verify the theoretical results.

Keywords: First order system least squares method, Helmholtz equation, high wave number, pollution error, stability, error estimate

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