

ADAPTIVE FINITE ELEMENT METHODS FOR AN OPTIMAL CONTROL PROBLEMS INVOLVING DIRAC MEASURES

ALEJANDRO ALLENDES, HARBIR ANTIL, ENRIQUE OTÁROLA, RICHARD RANKIN,
AND ABNER J. SALGADO

ABSTRACT. The purpose of this work is the design and analysis of a reliable and efficient a posteriori error estimator for the so-called pointwise tracking optimal control problem. This linear–quadratic optimal control problem entails the minimization of a cost functional that involves point evaluations of the state, thus leading to an adjoint problem with Dirac measures on the right hand side; control constraints are also considered. The proposed error estimator relies on a posteriori error estimates in the maximum norm for the state and in Muckenhoupt weighted Sobolev spaces for the adjoint state. We present an analysis that is valid for two and three-dimensional domains. We conclude by presenting several numerical experiments which reveal the competitive performance of adaptive methods based on the devised error estimator.

Keywords: pointwise tracking optimal control problem, dirac measures, a posteriori error analysis, adaptive finite elements, maximum norm, Muckenhoupt weights, weighted Sobolev spaces.

Mathematics Subject Classifications (2010): 49M25, 65K10, 65N15, 65N30, 65N50, 65Y20.

REFERENCES

- [1] J.P. Agnelli, E.M. Garau and P. Morin, A posteriori error estimates for elliptic problems with Dirac measure terms in weighted spaces. *ESAIM: Mathematical Modelling and Numerical Analysis*, 48:1557–1581, 2015.
- [2] H. Antil, E. Otárola and A.J. Salgado, Some applications of weighted norm inequalities to the analysis of optimal control problems. arXiv:1505.03919, 2015.
- [3] E. Dari, R.G. Durán and C. Padra, Maximum norm error estimators for three-dimensional elliptic problems. *SIAM Journal on Numerical Analysis*, 37(2):683–700, 2000.
- [4] R.H. Nochetto, Pointwise a posteriori error estimates for elliptic problems on highly graded meshes. *Mathematics of Computation*, 64(209):1–22, 1995.

DEPARTAMENTO DE MATEMÁTICA, UNIVERSIDAD TÉCNICA FEDERICO SANTA MARÍA, VALPARAÍSO, CHILE.
E-mail address: `alejandro.allendes@usm.cl`

DEPARTMENT OF MATHEMATICAL SCIENCES, GEORGE MASON UNIVERSITY, FAIRFAX, VA 22030, USA.
E-mail address: `hantil@gmu.edu`

DEPARTAMENTO DE MATEMÁTICA, UNIVERSIDAD TÉCNICA FEDERICO SANTA MARÍA, VALPARAÍSO, CHILE.
E-mail address: `enrique.otarola@usm.cl`

DEPARTAMENTO DE MATEMÁTICA, UNIVERSIDAD TÉCNICA FEDERICO SANTA MARÍA, VALPARAÍSO, CHILE.
E-mail address: `richard.rankin@usm.cl`

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TENNESSEE, KNOXVILLE, TN 37996, USA.
E-mail address: `asalgad1@utk.edu`