

# A PDE APPROACH TO SPACE–TIME FRACTIONAL PARABOLIC PROBLEMS

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**ABSTRACT.** We study solution techniques for parabolic equations with fractional diffusion and Caputo fractional time derivative, the latter being discretized and analyzed in a general Hilbert space setting. The spatial fractional diffusion is realized as the Dirichlet–to–Neumann map for a nonuniformly elliptic problem posed on a semi–infinite cylinder in one more spatial dimension. We write our evolution problem as a quasi-stationary elliptic problem with a dynamic boundary condition. We propose and analyze an implicit fully–discrete scheme: first–degree tensor product finite elements in space and an implicit finite difference discretization in time. We prove stability and error estimates for this scheme.

**Keywords:** Fractional derivatives and integrals, fractional diffusion, weighted Sobolev spaces, finite elements, stability, anisotropic estimates, fully-discrete methods.

**Mathematics Subject Classifications (2010):** 26A33, 65J08, 65M12, 65M15, 65M60, 65R10.

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