NUMERICAL SOLUTION OF 2D POPULATION BALANCE MODELS FOR WASTEWATER TREATMENT

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Abstract. Population Balance Models (PBMs) represent a powerful modelling framework for the description of the dynamics of properties that are characterized by distributions. This distribution of properties under transient conditions has been demonstrated in many chemical engineering applications. Modelling efforts of several current and future unit processes in Waste Water Treatment Plants (WWTPs) could potentially benefit from this framework, especially when distributed dynamics have a significant impact on the overall unit process performance. In these cases, current models that rely on average properties cannot sufficiently capture the true behaviour and even lead to completely wrong conclusions. Examples of distributed properties are bubble size, floc size, crystal size or granule size. In these cases, PBMs can be used to develop new knowledge that can be embedded in our current models to improve their predictive capability. Multiple properties can be distributed leading to multidimensional PBMs which are in nature integro PDEs.

The multidimensional pure aggregation population balance equation in a well mixed system is given as:

$$\frac{\partial f(t,\boldsymbol{x})}{\partial t} = \frac{1}{2}\int_0^{x} \beta(t,\boldsymbol{x}-\boldsymbol{x}',\boldsymbol{x}')f(t,\boldsymbol{x}-\boldsymbol{x}')f(t,\boldsymbol{x}')d\boldsymbol{x}' - \int_0^{\infty} \beta(t,\boldsymbol{x},\boldsymbol{x}')f(t,\ \boldsymbol{x})f(t,\boldsymbol{x}')d\boldsymbol{x}', \tag{1}$$

where $f(\boldsymbol{x},t)$ is the transient number density distribution and $\boldsymbol{x}$ is the particle state vector given in terms of some additive properties like mass or volume. The bold notations are used to denote vector quantities.

In this contribution, 2 numerical schemes for solving a 2D PBM on a rectangular grid, i.e. a finite volume scheme and a cell average technique, will be discussed.

Keywords: flocculation, coalescence, distributed properties

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