A MODIFICATION OF SCHIFFER'S CONJECTURE, AND A PROOF VIA FINITE ELEMENTS.

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Abstract. Approximations via conforming and non-conforming finite elements can be used to construct validated and computable bounds on eigenvalues for the Dirichlet Laplacian in certain domains. If these are to be used as part of a proof, care must be taken to ensure each step of the computation is validated and verifiable. In this talk we present a long-standing conjecture in spectral geometry, and its resolution using validated finite element computations.

Schiffer's conjecture states that if a bounded domain Ω in $\mathbb{R}^n$ has *any* nontrivial Neumann eigenfunction which is a constant on the boundary, then Ω must be a ball. This conjecture is open. A modification of Schiffer's conjecture is: for regular polygons of at least 5 sides, we can demonstrate the existence of a Neumann eigenfunction which is does not change sign on the boundary. In this talk, we provide a recent proof using finite element calculations for the regular pentagon. The strategy involves iteratively bounding eigenvalues for a sequence of polygonal subdomains of the triangle with mixed Dirichlet and Neumann boundary conditions. We use a learning algorithm to find and optimize this sequence of subdomains, and use non-conforming linear FEM to compute validated lower bounds for the lowest eigenvalue in each of these domains, [1]. The linear algebra is performed within interval arithmetic, [2]. This is joint work with Bartlomiej Siudeja and Ben Green, at U. Oregon.

Keywords: Eigenvalues, spectral geometry, non-conforming, validated numerics.

References

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