# AN ADER-TYPE SCHEME FOR A CLASS OF EQUATIONS ARISING FROM THE WATER-WAVE THEORY

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Abstract. In this work we propose a numerical strategy to solve a family of partial differential equations arising from the water-wave theory. These problems may contain four terms; a source which is an algebraic function of the solution, a convective part involving first order spatial derivatives of the solution, a diffusive part involving second order spatial derivatives and the transient part. Unlike partial differential equations of hyperbolic or parabolic type, where transient part is the time derivative of the solution, here transient part can contain mixed time and space derivatives. Examples of these type of partial differential equations are Saint-Venant equations and Bussinesq-Peregrine equations [1, 4].

In [8], authors proposed a globally implicit strategy to solve the Richard equation. In that case, transient terms consisted of algebraic equations of the solution. So motivated by this work, we propose a one-step finite volume method to deal with problems in which transient terms are differential operators. Here, a locally implicit formulation is investigated, which is based on the ADER philosophy first put forward by Toro et al. [6, 7]. The scheme is divided in three steps: i) a polynomial reconstruction of the data; ii) solutions to Generalized Riemann Problems (GRP); iii) the solution of differential problems. Note that steps i) and ii), are those of conventional ADER schemes, see [5, 2, 3]. Advantages of the present approach include the possibility to construct high-order approximations in both space and time, the differential problem can be non-linear and numerical strategies can be adopted to solve it, for which existing methodologies for hyperbolic problems can be applied. Convergence of the scheme is proved analytically and an empirical convergence rates assessment is carried out in order to illustrate the high space and time accuracy of the present scheme.

## References

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