

# APOSTERIORI ERROR ANALYSIS OF TIMESTEPPING SCHEMES FOR THE WAVE EQUATION

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ABSTRACT. A posteriori error estimates provide a rigorous foundation for the derivation of efficient adaptive algorithms for the approximation of solutions of partial differential equations. While the literature is rich as far as elliptic and parabolic equations are concerned, it is much less developed for the hyperbolic equations such as the wave equation. In this talk, I will review some of the “standard” a posteriori wave equation results, including those of [1] and [2], and present recent improvements and further developments to lower order Sobolev norms based on Baker’s Trick [3] for backward Euler schemes. Subsequent focus will be given to practically relevant methods such as Verlet, or Cosine, methods, a popular example of which is the Leap-frog method [4].

**Keywords:** a posteriori, error analysis, evolution PDE, hyperbolic, leapfrog, Newmark, time-stepping, Verlet, wave

**Mathematics Subject Classifications (2010):**

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