A ROBUST DPG METHOD FOR SINGULARLY PERTURBED REACTION-DIFFUSION PROBLEMS

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Abstract. We present and analyze a discontinuous Petrov-Galerkin method with optimal test functions for a reaction-dominated diffusion problem in two and three space dimensions. We start with an ultra-weak formulation that comprises parameters α, β to allow for general ε-dependent weightings of three field variables (ε being the small diffusion parameter). Specific values of α and β imply robustness of the method, that is, a quasi-optimal error estimate with a constant that is independent of ε. Moreover, these values lead to a norm for the field variables that is known to be balanced in ε for model problems with typical boundary layers. Several numerical examples underline our theoretical estimates and reveal stability of approximations even for very small ε.

Keywords: reaction-dominated diffusion, singularly perturbed problem, boundary layers, discontinuous Petrov-Galerkin method

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