PETROV–GALERKIN FINITE ELEMENT METHOD FOR FRACTIONAL CONVECTION-DIFFUSION EQUATIONS

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Abstract. We shall present a variational formulations of Petrov-Galerkin type for one-dimensional fractional boundary value problems involving either a Riemann-Liouville or Caputo derivative of order $\alpha \in (3/2, 2)$ in the leading term and both convection and potential terms. Such problems arise in mathematical modeling of asymmetric super-diffusion processes in heterogeneous media. The well-posedness of the formulations and sharp regularity pickup of the variational solutions will be discussed. A novel finite element method, which employs continuous piecewise linear finite elements and "shifted" fractional powers for the trial and test space, respectively, will be presented. The new approach has a number of distinct features. First, it allows deriving optimal error estimates in both L^2- and H^1-norms. Second, since the leading term of the stiffness matrix is diagonal for uniform meshes, it produces well conditioned linear algebraic systems. Further, in the case of Riemann-Liouville fractional derivative, we propose an enriched FEM, which improves the rate of convergence. Finally, some numerical results will be presented to verify the theoretical analysis and robustness of the numerical scheme.

Keywords: fractional convection-diffusion equation, variational formulation, finite element method, optimal error estimates

Mathematics Subject Classifications (2010): 65N30, 65N15

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