VOLUME INTEGRAL METHODS FOR THE HELMHOLTZ EQUATION WITH VARIABLE COEFFICIENTS.

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Abstract. We are interested in high order accurate numerical methods for the Helmholtz equation with variable coefficients

$$-\Delta u - \kappa^2(1 + q(\mathbf{x}))u = 0, \tag{1}$$

where the contrast function $q(\mathbf{x})$ represents the properties of a penetrable obstacle. In the penetrable case, as opposed to obstacle scattering case, a volume integral formulation is required. The corresponding volume integral equation is called the Lippmann-Schwinger equation [3]:

$$\rho(\mathbf{x}) + \kappa^2 q(\mathbf{x}) \int_{Supp(q)} \Phi(\mathbf{x}, \mathbf{y})\rho(\mathbf{y})\, d\mathbf{y} = -\kappa^2 q(\mathbf{x}) u^i(\mathbf{x}) \tag{2}$$

where the background Green's function is $\Phi(\mathbf{x}, \mathbf{y}) = \frac{i}{4} H_0^{(1)}(\kappa|\mathbf{x} - \mathbf{y}|)$. The integral equation is derived by representing the scattered field u^s as the convolution of the background Green's function for the exterior homogeneous medium with an unknown density ρ. The total field $u = u^s + u^i$ satisfies the equation (1) where u^i is a known incoming field.

In joint work with Leslie Greengard, Carlos Borges, and Sivaram Ambikasaran, we developed an automatically adaptive, high-order accurate discretization method that provides rapid access to arbitrary elements of the system matrix. For this, we used a level-restricted adaptive quad-tree. For kth order accuracy, on each leaf node, we approximate the unknown density ρ using a kth order polynomial approximation. Computing the matrix entries in the near field is complicated because of the weakly singular nature of the integrand. A principal novelty in our paper [1] is the use of precomputation, asymptotic analysis, and special function theory to reduce the computational cost so that only a few floating point operations are needed, in either the near or far field. The system itself was solved using a hierarchical fast solver (HODLR) [2], requiring $O(N^{3/2})$ work, where N is the total number of degrees of freedom.

Keywords: Acoustic scattering, electromagnetic scattering, penetrable media, fast direct solver, integral equation, Lippmann-Schwinger equation, high order accuracy, adaptivity

Mathematics Subject Classifications (2010): 65R20

References

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