MONOTONICITY–PRESERVING PERTURBATIONS OF RUNGE-KUTTA SCHEMES

I. HIGUERAS, D. KETCHESON, AND T.A. KOCSIS

Abstract. Exact solutions of some ordinary differential systems have qualitative properties, e.g., monotonicity, positivity, etc., which are relevant in the context of the problem. In these cases, it is convenient to preserve them numerically. For Runge-Kutta (RK) methods, the above qualitative properties can be ensured under stepsize restrictions given in terms of the radius of absolute monotonicity, also known as SSP or Kraaijevanger's coefficient.

However, some well known RK methods have a trivial SSP coefficient and, therefore, monotonicity cannot be ensured. For one of these methods, in [6] a second spatial discretization of the PDE is used wherever a negative coefficient appears in the time integration method; this process can be interpreted as a perturbation of the original RK method [2]. During the last years, several authors have studied perturbed RK methods (also known as downwind RK methods) to find high order schemes with optimal SSP coefficients (see, e.g., [5]).

But monotonicity preservation is not the only numerical property of interest in applications, and one may wish to use a particular method that has small or zero SSP coefficient. In this case, we can use a perturbation of the scheme that ensures a larger monotonicity–preserving time step.

In this talk we show some results on optimal monotonicity–preserving perturbations of a given explicit RK method. The presentation is based on the paper [5].

This framework can also be used to explain the qualitative correct solutions obtained in the context of positivity when some problems are integrated with explicit RK schemes that have trivial SSP coefficients (or IMEX RK methods that have trivial regions of absolute monotonicity) [1, 3].

Keywords: Strong stability preserving, monotonicity, positivity, Runge–Kutta methods, IMEX Runge–Kutta methods, downwind Runge–Kutta methods

Mathematics Subject Classifications (2010): 65L06, 65L20, 65M20

References

[1] R. Donat, I. Higueras, and A. Martínez-Gavara. On stability issues for IMEX schemes applied to hyperbolic equations with stiff reaction terms. *Mathematics of Computation*, 80:2097–2126, 2011.

[2] I. Higueras. Strong Stability for additive Runge-Kutta methods. SIAM *Journal on Numerical Analysis*, 44(4):1735–1758, 2006.

[3] I. Higueras. Positivity properties for the classical fourth order Runge-Kutta method. Monografías de la Real Academia de Ciencias de Zaragoza, 33:125–139, 2010.

[4] I. Higueras, D. Ketcheson, and T.A. Kocsis. Optimal perturbations of Runge-Kutta methods arXiv:1505.04024, 2015.

[5] S. J. Ruuth, and R. J. Spiteri. High-order strong-stability-preserving Runge-Kutta methods with downwindbiased spatial discretizations. SIAM *Journal on Numerical Analysis*, 42:974–996, 2004.

[6] C. W. Shu, and S. Osher. Efficient implementation of essentially non-oscillatory shock-capturing schemes. *Journal of Computational Physics*, 77(2):439–471, 1988.

Public University of Navarre, Pamplona, Spain
E-mail address: `higueras@unavarra.es`

King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia
E-mail address: `david.ketcheson@kaust.edu.sa`

Szechenyi Istvan University, Györ, Hungary
E-mail address: `katihi@sze.hu`