HYBRID NUMERICAL-ASYMPTOTIC BOUNDARY ELEMENT METHODS FOR HIGH FREQUENCY WAVE SCATTERING

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Abstract. The boundary element method (BEM) is a well-established tool for solving time-harmonic acoustic and electromagnetic wave scattering problems. However, conventional BEMs, with piecewise polynomial approximation spaces, suffer from the restriction that a fixed number of degrees of freedom is required per wavelength in order to represent the oscillatory solution, leading to high computational cost in the high frequency regime where the scatterer is large compared to the wavelength.

The *hybrid numerical-asymptotic* (HNA) approach (recently reviewed in [2]) aims to dramatically reduce the number of degrees of freedom required at high frequencies, by enriching the BEM approximation space with oscillatory functions, chosen using partial knowledge of the high frequency asymptotic behaviour of the solution (as described e.g. by the Geometrical Theory of Diffraction [1]). The HNA approach has been successfully applied to a range of problems (mostly but not exclusively two-dimensional) including scattering by sound-soft smooth convex obstacles [4], convex [8] and nonconvex [3] polygons and planar screens [7], [2, §7.6]. In many cases these methods can achieve the "holy grail" of frequency-independent computational cost, provided that the numerical integration required for practical implementation is carried out using appropriate oscillatory quadrature techniques [7].

In my talk I will outline the basic HNA methodology, and will give an overview of current research into HNA methods for nonconvex scatterers [3, 6], 3D scatterers [2, §7.6], and transmission problems [5], where complicated multiple scattering and shadowing effects present interesting analytical and numerical challenges. This talk represents joint work with Simon Chandler-Wilde, Stephen Langdon, Ashley Twigger, Samuel Groth (University of Reading), Emile Parolin (University of Oxford) and Markus Melenk (TU Vienna).

Keywords: Helmholtz equation, high frequency scattering, boundary element method.

Mathematics Subject Classifications (2010): 65N12, 65N38, 65R20.

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