CONVERGENCE ORDER ANALYSES OF ADI SPLITTING SCHEMES FOR SEMILINEAR PARABOLIC PDES

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Abstract. Splitting schemes are widely used for time integration of partial differential equations (PDEs). The idea is to split the original PDE into subproblems and solve them separately. For example the diffusion may be treated separately from the reaction in a reaction-diffusion equation, further, for the diffusion, the different spatial dimensions may be treated one at a time. Efficiency may be gained whenever the subproblems are simpler to solve than the original problem.

We consider the general framework of semilinear dissipative evolution equations. That is, we assume that the vector field of the evolution equation is the sum of two unbounded, dissipative operators, one of which may be nonlinear. This setting includes many parabolic PDEs, e.g. reaction-diffusion equations with a linear diffusion and a nonlinear reaction term. We perform convergence order analyses of splitting methods for these evolution equations.

Our analyses focus on splitting methods that exhibit excellent stability properties when applied to dissipative evolution equations. The emphasis is put on ADI splitting methods and most prominently the Peaceman–Rachford and Douglas–Rachford schemes. Further, in contrast to what is usually done, we do not assume Lipschitz continuity of the nonlinear flow. We also take the analysis further by combining the temporal (splitting) discretizations with spatial discretizations and prove simultaneous space-time convergence orders.

Beyond the convergence analyses we will also present applications found among parabolic PDEs. Numerical examples are given to illustrate our results.

Keywords: Splitting schemes, convergence order analysis, semilinear parabolic PDEs, semilinear dissipative evolution equations, ADI schemes.

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