

ON SPECTRAL DIFFERENTIATION ON THE DISK

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ABSTRACT. We discuss computational and analytical aspects of differentiation of functions defined on the unit disk represented by their generalized Fourier expansions with respect to families of Zernike orthogonal polynomials. In particular, we report on the degree of regularity functions require for the validity of an infinite matrix-vector product formula expressing the coefficients of the expansion of a derivative in terms of the coefficients of the expansion of the original function and show a counterexample. Many of the arguments are very much in the spirit of the classical treatment of orthogonal polynomial expansions.

Some analytical and computational consequences of the main result as well as a brief discussion on the non-isotropic weighted Sobolev-type function spaces naturally involved in this problem are included. We exhibit some numerical examples illustrating how some of the derived results and identities inform the design of tau-type spectral methods on the disk.

Keywords: Zernike polynomials, spectral methods, disk, Sobolev spaces.

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