DERIVATION AND VALIDATION OF IMPEDANCE TRANSMISSION CONDITIONS FOR THE ELECTRIC POTENTIAL ACROSS A HIGHLY CONDUCTIVE CASING

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Abstract. Borehole resistivity measurements are a common procedure when trying to obtain a better characterization of the Earth's subsurface. The possible risk of having borehole collapses makes the employment of a casing very suitable for this type of scenarios. Such casing protects the borehole but it also highly complicates the numerical analysis due to its thinness and large contrast between the conductivities of the casing and the rock formations.

In this work, motivated by realistic configurations, we consider the conductivity of the casing to have the form $\sigma_c \approx \varepsilon^{-3}$ when ε denotes the thickness of the casing. In this framework, our aim is to derive Impedance Transmission Conditions (ITCs) for the electromagnetic field across such a casing. As a first approach we derive ITCs for the electric potential.

Similar studies regarding the derivation of ITCs for Electromagnetism have already been done. In [1], the authors derive ITCs for electromagnetic models and in particular, apply them to biological cells, where there is a contrast between the complex wave numbers of the different layers that compose the cells. Also, in [2], the authors derive ITCs for eddy current models with a conductivity parameter on a thin sheet of the form $\sigma \approx \varepsilon^{-2}$.

This work can be seen as the continuation of [3], where we consider a transmission problem for the static case of the electric potential (u), $\operatorname{div}(\sigma \nabla u) = f$, set in an axisymmetric borehole shaped domain. This domain is composed of three different subdomains, the interior part of the borehole, the rock formations and the metallic casing. Here σ represents the conductivity and f represents the right hand side.

In this framework we address the issue of ITCs for u (as $\varepsilon \to 0$) using two different approaches. The first one consists in deriving ITCs across the casing itself, whereas the second approach tackles the problem by deriving ITCs on an artificial interface located in the middle of the casing.

In [3] we derive different models for the two considered approaches and we numerically assess them with a finite element method implementation. Then we perform a comparison on these models by showing the advantages and drawbacks of each model. Finally, we show an application to a borehole through-casing resistivity measurement scenario. This work delivers both existence and uniqueness proofs, along with stability results and error estimates, leading to convergence of each approximate model. **Keywords**: Impedance Conditions, Electric Potential, Borehole, Casing, Resistivity, Conductivity.

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