

# ON THE GLOBAL OPTIMALITY OF THE REGULAR PENTAGON FOR THE DIRICHLET EIGENVALUE PROBLEM

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ABSTRACT. In this paper, we present a novel numerical strategy to study the Szegő-Pólya conjecture on 5-gons which states that the minimizer of the first Dirichlet eigenvalue over all 5-gons of given area is the regular pentagon. This conjecture is still open. We will describe known results on this conjecture. We will then describe a novel coupled Bayesian optimization and Finite Element method (via conforming and non-conforming piecewise linear functions) to study this problem. Finally, we will provide several results.

**Keywords:** Szegő-Pólya conjecture, first Dirichlet eigenvalue, Bayesian optimization, Finite element method.

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