A COUPLING STRATEGY FOR NONLOCAL AND LOCAL DIFFUSION MODELS WITH MIXED VOLUME CONSTRAINTS AND BOUNDARY CONDITIONS

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Abstract. The use of nonlocal models in science and engineering applications has been steadily increasing over the past decade. The ability of nonlocal theories to accurately capture effects that are difficult or impossible to represent by local Partial Differential Equation (PDE) models motivates and drives the interest in this type of simulations.

However, the improved accuracy of nonlocal models comes at the price of a significant increase in computational costs compared to, e.g., traditional PDEs. In particular, a complete nonlocal simulation remains computationally untenable for many science and engineering applications. As a result, it is important to develop Local–to–Nonlocal coupling strategies, which aim to combine the accuracy of nonlocal models with the computational efficiency of PDEs. The basic idea is to use the more efficient PDE model everywhere except in those parts of the domain that require the improved accuracy of the nonlocal model.

We develop and analyze an optimization–based method for the coupling of nonlocal and local diffusion problems with mixed volume constraints and boundary conditions. The approach, introduced in [1], formulates the coupling as a control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the nonlocal and local domains, and the controls are virtual volume constraints and boundary conditions. We prove that the resulting optimization problem is well–posed and discuss its implementation using Sandia's agile software components toolkit.

Numerical results for nonlocal diffusion in three–dimensions illustrate key properties of the optimization–based coupling method; these numerical tests provide the groundwork for the development of efficient and effective engineering analysis tools.

Keywords: Nonlocal models, coupling method, optimization, nonlocal vector calculus, mixed boundary conditions, nonlocal diffusion.

References

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