A COMPRESSIBLE TWO-LAYER MODEL FOR TRANSIENT TWO-PHASE FLOWS IN PIPES

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Abstract. This work is dedicated to the modelling of air/water flows in pipes. As a first step, a new 1D model is proposed to deal with the stratified regime. The starting point is the 2D Isentropic Euler set of equations for phase k where the classical hydrostatic assumption is made for water ($k = 1$) while the gravitational effects are neglected for air ($k = 2$). The main difference with the models issued from the classical literature is that water as well as air are assumed compressible. In that framework, an averaging process across the layer depth results in a 5 equation system where the hydrostatic constraint has been introduced in an original manner. Closure laws for interfacial terms and source terms are provided following an *entropy inequality* and the overall model is hyperbolic with non conservative terms. Therefore, *the definition and uniqueness of jump conditions* is studied carefully and acquired. Regarding the Riemann problem associated with the homogeneous problem, the nature of characteristic fields and the corresponding Riemann invariants are exhibited. Positivity is also obtained for heights h_k and densities $\rho_k,\ k = 1, 2$. Finally, numerical tests using the Rusanov scheme illustrate the ability of the model to deal with discontinuous solutions such as shock waves and contact discontinuities. Denoting respectively u_k and $P_k(\rho_k)$ the velocity and the pressure of phase k, g the gravitational acceleration, $(\lambda_p, \lambda_m, \lambda_u)$ some positive functions including relaxation time-scales and defining $\Psi_k(\rho_k) = \frac{P_k}{\rho_k^2}$, the closed system reads for $k = 1, 2$:

$$\frac{\partial h_1}{\partial t} + u_2 \frac{\partial h_1}{\partial x} = \lambda_p (P_1 - \rho_1 g \frac{h_1}{2} - P_2), \tag{1a}$$

$$\frac{\partial h_k \rho_k}{\partial t} + \frac{\partial h_k \rho_k u_k}{\partial x} = (-1)^k \lambda_m \Big((\frac{P_1 + \rho_1 g \frac{h_1}{2}}{\rho_1} + \Psi_1) - (\frac{P_2}{\rho_2} + \Psi_2) + \frac{u_2^2 - u_1^2}{2} \Big), \tag{1b}$$

$$\frac{\partial h_k \rho_k u_k}{\partial t} + \frac{\partial h_k (\rho_k u_k^2 + P_k)}{\partial x} - (P_1 - \rho_1 g \frac{h_1}{2}) \frac{\partial h_k}{\partial x} = (-1)^k \lambda_u (u_1 - u_2). \tag{1c}$$

Keywords: two-phase flow, free-surface flow, hyperbolic system, non-conservative terms

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