MULTIDIMENSIONAL, SELF-SIMILAR, STRONGLY-INTERACTING, CONSISTENT (MUSIC) RIEMANN SOLVERS—APPLICATIONS TO DIVERGENCE-FREE MHD AND ALE SCHEMES

DINSHAW S. BALSARA

ABSTRACT. Large-scale, multidimensional flow simulations are now commonplace and there is a considerable interest in very accurate algorithms for such simulations. Introducing true multidimensionality in such algorithms is very valuable for a complete representation of the physics of the problem. Reconstruction strategies for hyperbolic PDEs (WENO, DG, PNPM, MOOD) are already fully multidimensional as are methods for their temporal update (RK, ADER). The majority of Riemann solvers are still one-dimensional. The present talk describes the design of multidimensional Riemann solvers and their applicability to higher order schemes.

Such multidimensional Riemann solvers act at the vertices of the mesh, where the multidimensional flow structure becomes visible to the Riemann solver. Instead of two input states, the input states consist of states from all the zones that meet at that vertex. At any zone interface that separates two states, a one dimensional Riemann problem emanates, as always. However, at any vertex, all the adjacent one-dimensional Riemann problems interact to form a strongly interacting state. The strongly interacting state evolves self-similarly in spacetime. By evolving the structure of the strongly interacting state in a set of self-similar variables we show that the structure of the strongly interacting state can be elucidated. Self-similarity is crucially important in the development of multidimensional Riemann solvers. This has prompted the name of MuSIC Riemann solvers, where MuSIC stands for "Multidimensional, Self-similar, strongly-Interacting, Consistent".

Keywords: Multidimensional Riemann solvers; Higher order schemes; MHD; ALE

UNIVERSITY OF NOTRE DAME
E-mail address: dbalsara@nd.edu